Multivariable Nonlinear and Adaptive Control of a Distillation Column

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In chemical process control, the processes typically exhibit nonlinear behavior. In spite of this, linear feedback laws often perform quite well as long as the process is being controlled at a fixed operating point. Linear controller designs based on linearized process models are therefore commonly used. The nonlinear process dynamics can be taken into account implicitly by treating them as part of the model uncertainties.

If the nonlinearity of the process is too severe for a (single) linear control law to perform satisfactorily, one possibility is to describe the process by a set of different linear models, each valid at a given operating point. One can then derive a different linear feedback law for each operating point and use the feedback law corresponding to the prevailing operating point (Shamma and Athans, 1991). A potential difficulty with this approach is to decide which model and feedback law to use between the given set of operating points.

Another possibility is to describe the process by a nonlinear model valid over the whole operating range of interest. The main difficulty with this approach is to find a suitable nonlinear model. Models derived from physical principles are in general much too complex for controller design, and therefore simpler models that capture the nonlinearities are required. If a nonlinear model is available, one can design a nonlinear controller based on this model for the whole operating range (Slotine and Li, 1991). However, it is also possible to use a linear controller whose parameters are updated at each sampling instant on the basis of a linearized form of the nonlinear model.

For processes with poorly known or slowly time-varying dynamics, adaptive and self-tuning control are widely accepted approaches (Åström and Wittenmark, 1989). When applying adaptive control to nonlinear processes, care should be taken to handle the nonlinearities properly. Adaptive controllers are usually based on linear process models and linear design methods. Although adaptive controllers have the property of adapting their behavior to changing process properties, it is, however, somewhat unsatisfactory to not consider the fact that the process is nonlinear, and to let the adaptive controller retune itself (and the parameters of the linear model) whenever the operating point is changed. As above, possible remedies (including the same problems and difficulties) are to let the algorithm employ several linear process models, each valid at a given operating point, or a nonlinear process model valid over the whole operating range.

In this article, multivariable nonlinear and adaptive control is applied to a binary distillation column (Waller et al., 1988). Previous investigations (Waller et al., 1988; Sandelin et al. 1991; Häggblom, 1993) have shown that the process has nonlinear dynamics. Although linear controllers can be used to control the process at any given operating point, it is difficult to control the process satisfactorily over the whole operating range of interest using a fixed linear control law. Moreover, experimental data show that the dynamic behavior of the process may change with time even at the same operating point (Sandelin et al., 1991). It is therefore well motivated to apply nonlinear and adaptive control strategies to the process.

The nonlinear model used in this study is determined as follows. From previous studies, experimentally determined linear process models at various operating points are available (Sandelin et al., 1991). From these models one can identify the parameters which are most subject to change when the operating point is varied. These parameters are modeled to depend linearly on the operating point. This procedure results in a bilinear model, which is fitted to the linear models at the respective operating points. The bilinear model captures the nonlinear behavior of the process satisfactorily in the operating range of interest.

The control law used in the study is a simple robust tuning method for multivariable PID controllers that has been proposed in the literature (Davison, 1976; Penttinen and Koivo, 1980; Peltomaa and Koivo, 1983; Tanttu, 1987). Because the method is based on a linear process model, the controller parameters are updated at each sampling instant on the basis of a linearized form of the bilinear model. As the bilinear model is linear in the parameters, these can also be estimated on-line by standard parameter estimation methods. This results in an adaptive controller.

In this article the modeling of the process and the controller tuning are described. Experimental results obtained with the following control strategies are presented: linear feedback control with fixed parameters, feedback control based on a bilinear model with fixed parameters, and adaptive control based on a bilinear model.

Process Modeling

The process studied is a 15-plate pilot-plant distillation column described in Waller et al. (1988). The controlled process variables are the temperatures T_4 and T_{14} on plates 4 and 14, respectively (counting from the top). The manipulated variables used in this study are the reflux flow (L) and the steam flow to the reboiler (V).

The set point of T_4 is held fixed at about 79.5°C, whereas the operating range of interest for T_{14} is $86°C \leq T_{14} \leq 89°C$. The column exhibits a nonlinear dynamic behavior, and it is therefore difficult to achieve satisfactory control performance over the whole operating range using a fixed linear control law (Waller et al., 1988; Sandelin et al., 1991).

The nonlinear nature of the process is evident from experimentally identified models obtained at two different operating points (Sandelin et al., 1991). The transfer functions corresponding to the lower temperature level $T_{14} = 86°C$ (operating point A) and the higher temperature level $T_{14} = 89°C$ (operating point B), respectively, are:

$$\begin{pmatrix} \Delta T_4 \\ \Delta T_{14} \end{pmatrix} = \begin{pmatrix} \dfrac{-0.038e^{-0.5s}}{8.2s+1} & \dfrac{0.049e^{-0.5s}}{14.2s+1} \\ \dfrac{-0.187e^{-1.5s}}{7.8s+1} & \dfrac{0.48e^{-0.5s}}{13.4s+1} \end{pmatrix} \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} \quad \text{(operating point A)} \tag{1a}$$

$$\begin{pmatrix} \Delta T_4 \\ \Delta T_{14} \end{pmatrix} = \begin{pmatrix} \dfrac{-0.036e^{-0.5s}}{7.5s+1} & \dfrac{0.052e^{-0.5s}}{12.6s+1} \\ \dfrac{-0.247e^{-1.5s}}{8.2s+1} & \dfrac{0.83e^{-0.5s}}{7.5s+1} \end{pmatrix} \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} \quad \text{(operating point B)} \tag{1b}$$

Here Δ denotes the deviations from the respective steady-state values. The unit for the time constants and delays is minutes, for the temperatures it is °C, and for L and V, kg/h.

The models (Eqs. 1) were sampled with the sampling interval 0.5 min, assuming piecewise constant control inputs L and V over the sampling interval. The sampled discrete models are:

$$\begin{aligned} \Delta T_4(k) &= \frac{b_{11}}{1-a_{11}q^{-1}} \Delta L(k-2) + \frac{b_{12}}{1-a_{12}q^{-1}} \Delta V(k-2) \\ \Delta T_{14}(k) &= \frac{b_{21}}{1-a_{21}q^{-1}} \Delta L(k-4) + \frac{b_{22}}{1-a_{22}q^{-1}} \Delta V(k-2) \end{aligned} \tag{2}$$

Table 1. Parameters of Model (Eq. 2)

	Operating Point A	Operating Point B
a_{11}	0.941	0.936
a_{12}	0.965	0.961
b_{11}	-0.00225	-0.00232
b_{12}	0.00170	0.00202
a_{21}	0.938	0.941
a_{22}	0.963	0.936
b_{21}	-0.0116	-0.0146
b_{22}	0.0176	0.0535

where q^{-1} is the backward shift operator ($q^{-1}x(k) = x(k-1)$). The parameters of the discrete model (Eq. 2) corresponding to the models (Eqs. 1) at the operating points A and B are shown in Table 1.

It is desired to model the process by a simple nonlinear model. Therefore, a model with the same structure as that in Eq. 2 is selected, but with parameters that depend linearly on the operating point defined by the temperatures. From Table 1 it is evident that a large part of the nonlinearity is captured by the parameters b_{21} and b_{22}. It is also seen that $a_{11} \approx a_{12}$ and $a_{21} \approx a_{22}$. This leads to the bilinear model structure:

$$\begin{aligned} y_1(k+1) &= a_1 y_1(k) + b_{11}u_1(k-1) + b_{12}u_2(k-1) + c_1 \\ y_2(k+1) &= a_2 y_2(k) + [b_{210} + b_{211}y_2(k)]u_1(k-3) \\ &\quad + [b_{220} + b_{221}y_2(k)]u_2(k-1) + c_2 \end{aligned} \tag{3}$$

Here y_1, y_2, u_1 and u_2 denote deviations of T_4, T_{14}, L and V, respectively, from fixed reference values. The bias terms c_i are introduced to account for disturbances, as well as any incompatibilities in the respective reference values of the inputs and outputs. When the model (Eq. 3) is fitted to the sampled form (Eq. 2) of the models in Eq. 1 at the respective operating points, the parameter values given in Table 2 are obtained. Since the bilinear model (Eq. 3) is linear in the parameters, standard linear parameter estimation procedures, such as least-squares estimation, can be used in adaptive control algorithms based on this model structure.

Controller Tuning

The control algorithm used in this study is a simple method for tuning multivariable PID controllers. The procedure was originally introduced by Davison (1976) for tuning multivariable PI controllers based on step response data. The method was modified by Penttinen and Koivo (1980), and adapted to

Table 2. Parameters of Model (Eq. 3)

a_1	0.951
b_{11}	-0.00229
b_{12}	0.00186
a_2	0.945
b_{210}	-0.0131
b_{211}	-0.001
b_{220}	0.0356
b_{221}	0.012

the discrete-time case by Peltomaa and Koivo (1983). Tanttu (1987) applied the tuning procedure in a self-tuning controller.

Here we follow the approach of Tanttu (1987). Consider an m-input/m-output process which can be described by the model:

$$y(k+1)=A_1y(k)+\cdots+A_{n_a}y(k+1-n_a)+B_du(k-d)+\cdots+B_{n_b}u(k-n_b) \quad (4)$$

The controller is assumed to be a discrete-time PID controller:

$$u(k)=K_Pe(k)+K_I\sum_{i=0}^{k-1}e(i)+K_D[e(k)-e(k-1)] \quad (5)$$

where $e(k)$ is the control error

$$e(k)=y_r(k)-y(k) \quad (6)$$

between the reference value y_r and the output y.

The controller tuning matrices are represented in the form:

$$K_P=K_{P0}\,\mathrm{diag}(p_1,p_2,\ldots,p_m),\quad 0<p_i\leq 1,\quad i=1,\ldots,m \quad (7a)$$

$$K_I=K_{I0}\,\mathrm{diag}(\epsilon_1,\epsilon_2,\ldots,\epsilon_m),\quad \epsilon_i>0,\quad i=1,\ldots,m \quad (7b)$$

$$K_D=K_{D0}\,\mathrm{diag}(\delta_1,\delta_2,\ldots,\delta_m),\quad \delta_i>0,\quad i=1,\ldots,m \quad (7c)$$

where K_{P0}, K_{I0}, and K_{D0} are rough tuning gains, and p_i, ϵ_i, and δ_i are fine-tuning parameters. The idea is that the rough tuning gains can be determined as simple functions of the process characteristics, and the fine-tuning parameters can then be used for fine adjustment of the closed-loop response.

For the proportional part of the controller, a suitable choice for the rough tuning gain is:

$$K_{P0}=B_d^{-1} \quad (8)$$

assuming that B_d is invertible (Tanttu, 1987). A good choice for the integral part is the inverse of the steady-state gain matrix (Davison, 1976), that is,

$$K_{I0}=\left(\sum_{i=d}^{n_b}B_i\right)^{-1}\left(I-\sum_{i=1}^{n_a}A_i\right) \quad (9)$$

Finally, for the rough tuning matrix of the derivative part, the choice:

$$K_{D0}=K_{P0} \quad (10)$$

has been proposed (Koivo, 1980). In the derivative part, very small fine-tuning parameters δ_i are recommended since large gains in the derivative part may cause oscillations or even instability in the closed-loop system.

In adaptive control, the rough tuning of the controller is performed on-line by using the estimated process model to compute the tuning matrices according to Eqs. 8-10 (Tanttu, 1987). When the process model is nonlinear, as in Eq. 3, the PID tuning procedure based on Eqs. 7-10 can still be used by applying a linearized version of the process model at each sampling instant.

Experimental Results

Controllers based on the bilinear model (Eq. 3) and the multivariable PID controller tuning approach described earlier were applied to the pilot-plant distillation column. Besides Eq. 3, other model structures were also tested, but these did not provide any improvements in control performance. In all experiments, the fine-tuning parameters:

$$p_i=0.2,\quad \epsilon_i=0.2,\quad \delta_i=0.01,\quad i=1,2$$

were used. These values were found by trial and error.

In the first experiment, which was made in order to demonstrate the nonlinear behavior of the column, a fixed-parameter controller based on a linear process model was used. The parameters of this model were taken as the mean of the parameters of the two experimentally identified models A and B. The distillation column was subjected to set point changes in T_{14} within the interval 86°C ≤ T_{14} ≤ 90°C. This experiment is illustrated in Figure 1. It can be seen that T_4 is not much

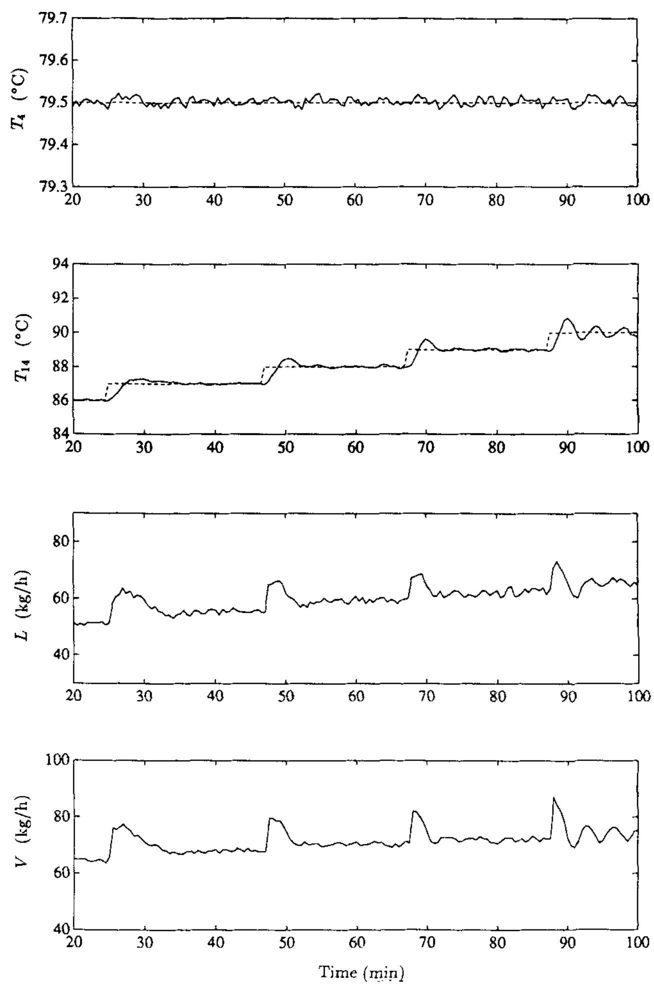

Figure 1. Linear feedback control with fixed parameters.

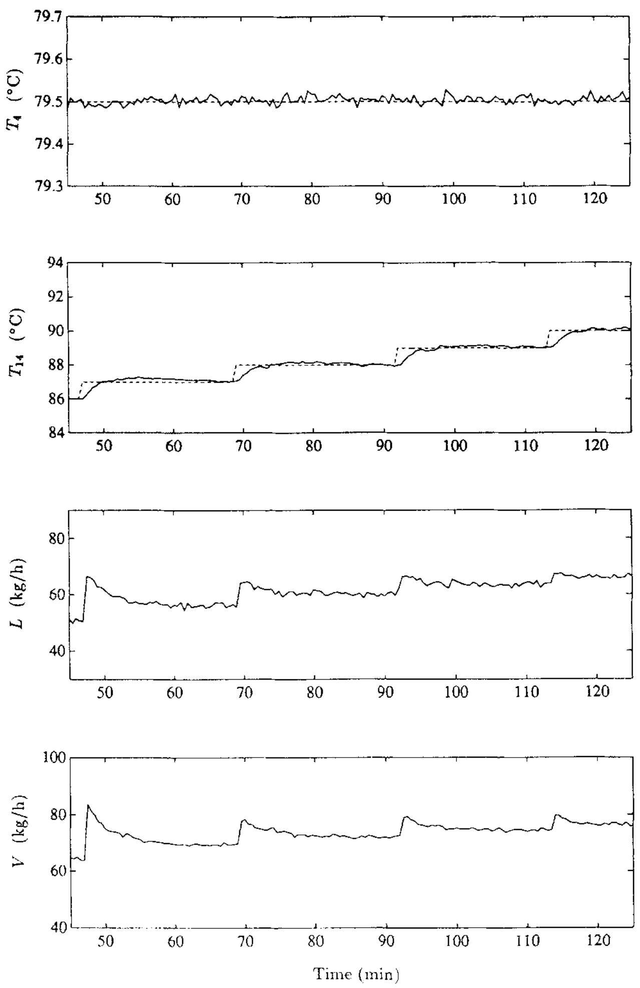

Figure 2. Feedback control based on a bilinear model with fixed parameters.

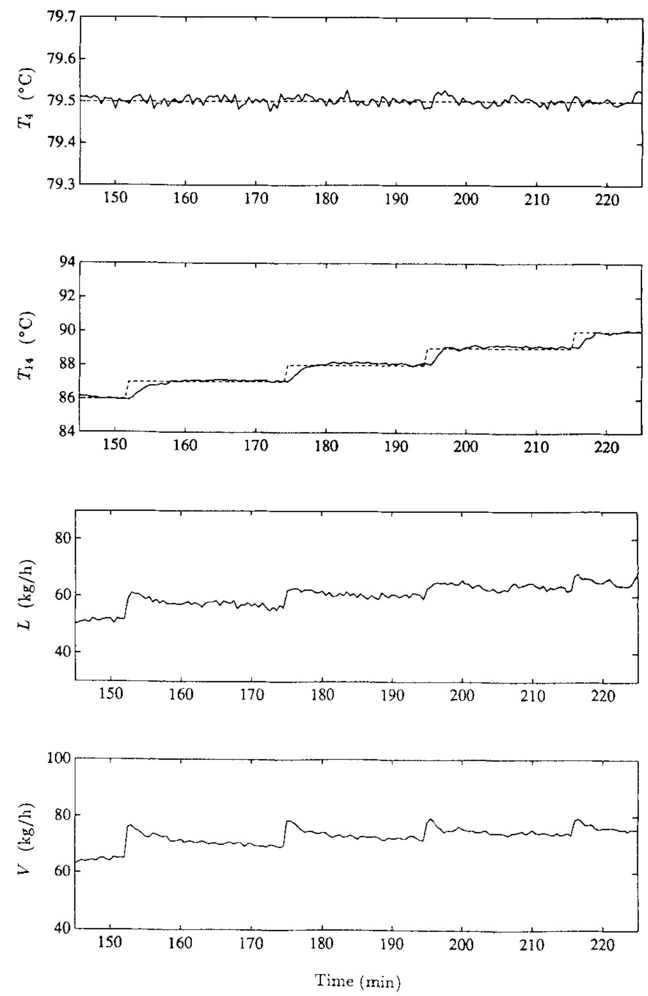

Figure 3. Adaptive control based on a bilinear model.

affected by the set point changes, that is, the decoupling properties of the control system seem to be good. As in earlier investigations (Waller et al., 1988), T_{14} is not satisfactory controlled over the whole operating range of interest.

In the second experiment, a controller based on the bilinear model (Eq. 3) was applied to the column. At each sampling instant, the controller parameters were calculated according to Eqs. 8-10 based on a linearized form of the bilinear model (Eq. 3). A similar sequence of set point changes as in the previous experiment was made. The result is shown in Figure 2. The responses obtained with this controller are superior to those in the previous experiment and uniform over the whole operating range.

In the third experiment, an adaptive controller based on the bilinear model (Eq. 3) was applied. The parameters of Eq. 3 were estimated recursively using the method of least squares. The adaptive control algorithm was initiated with the parameters given in Table 2 and the covariance matrix $P(0)=I$. A constant forgetting factor $\lambda=0.99$ was used. In order to improve the parameter estimates, a sequence of set point changes in T_4 and T_{14} were applied initially. After this, a similar experiment as in Figure 2 was carried out. The process responses are shown in Figure 3 and the parameter estimates in Figure 4. As Figure 3 shows, the responses are similar to those obtained without adaptation (Figure 2). However, the adaptive control law can cope with changes in the process behavior due to disturbances (such as changes in the feed), or changes in general that occur over a longer time range.

Conclusions

Nonlinear and adaptive control has been applied to a distillation process. Experimental process information in terms of linear process models valid at different operating points was used to obtain a bilinear process model. Since experimental process information is often in practice available as linear models at different operating points, the modeling procedure is likely to be useful for process control in general.

The process was controlled by a discrete-time multiinput/multioutput PID controller, whose model-dependent parameters were calculated at each sampling instant from a linearized form of the bilinear model. In this way, uniform performance was obtained over the whole operating range of interest. A similar controller with fixed parameters did not perform satisfactorily in this respect.

Because the bilinear model is linear in the parameters, these

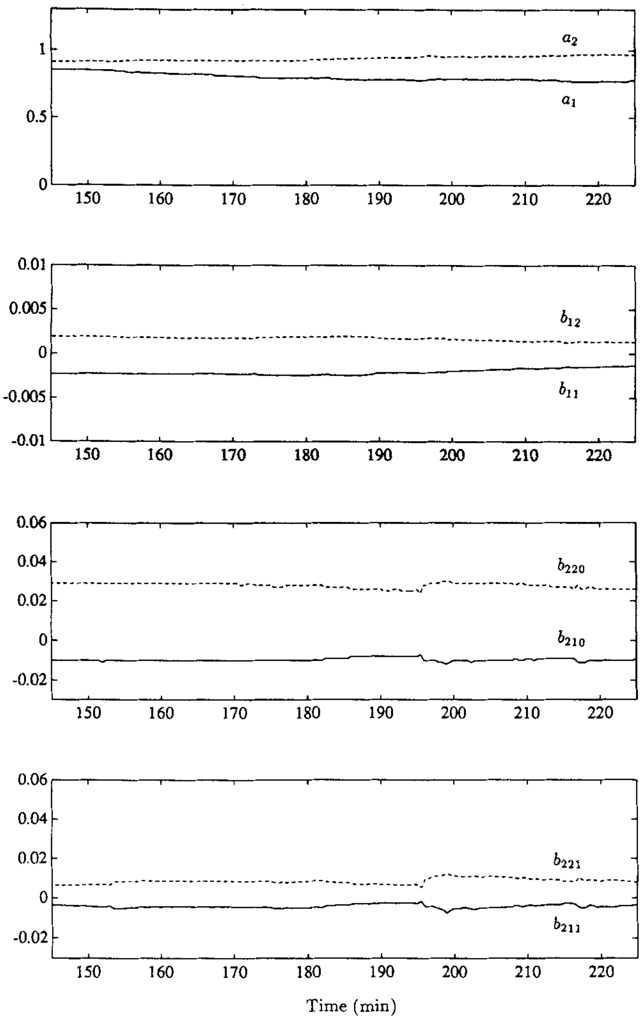

Figure 4. Parameter estimates for the experiment in Figure 3.

could also be estimated on-line by standard parameter estimation methods. The resulting adaptive controller, based on the same design procedure as the previous controller, also performed satisfactorily over the whole operating range.

Acknowledgment

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